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The use of a torsion machine to measure the shear strength and modulus of unidirectional carbon fibre reinforced plastic composites

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A torsion apparatus, in which a solid rod specimen is subjected to a shear stress field only, has been used to measure the shear modulus and strength of unidirectional carbon fibre reinforced plastics. Because of the absence of tensile and compressive forces, a more accurate value of the shear strength is obtained than from a test such as the short beam shear test. The shear strength is calculated allowing for the non-linear nature of the shear stress-strain characteristic. For type 2 treated fibre the shear modulus is found to increase rapidly with fibre volume loading, in reasonable agreement with the micromechanical theory of Heaton. For type 2 untreated and type 1 treated fibre composites, a slightly less rapid increase in shear modulus is noted. Results for type 1 untreated fibre composites increase with volume loading but are below both the other results and the theoretical curve. The shear strength of composite materials made from type 2 treated fibre is greater than that of the pure resin, and has a maximum for about 50% volume of fibre. For type 1 and untreated carbon fibres the shear strength decreases with increasing volume loading. By using the concepts of fracture mechanics and assuming that the bond between type 2 treated fibre and resin is completely effective, so that failure starts in the matrix, it is possible to give a plausible explanation of the shear strength results. The shear modulus, but not the shear strength, can be measured accurately, using either square or circular cross-section specimens.

1. Introduction

The interlaminar shear strength of a unidirectional composite as measured by the short-beam shear test is frequently used to indicate the degree of bonding between the fibre and matrix. However, because of the complex system of stresses in this test, failure is usually initiated by compression, as shown by Hancox and Wells [1], and influenced by the tensile strength of the fibre. A more reliable way of measuring the interlaminar shear strength, and also the shear modulus, is to twist a solid cylindrical specimen, one end of which is free to move axially, and note the torque as a function of the angle of rotation. The stress field is not uniform across the rod, but is one of shear only. In addition, Reynolds and Hancox [2] have recently shown that the strength and modulus results may be analysed using the concepts of fracture mechanics, and a parameter obtained which indicates the degree of bonding between the fibre and resin.

2. Experimental work

The torsion rig was designed by Bowes at Harwell, and full details of the design and construction will be given elsewhere [3]. A cylindrical specimen with square cross-section ends is gripped between two accurately aligned self-centring chucks. One chuck is coupled to a variable speed electric motor and can rotate clockwise or anticlockwise, while the other is free to rotate and move longitudinally, so that as the specimen is twisted no tensile stress is built up in it. The torque transmitted by the specimen is measured by means of an arm attached to the freely-rotating chuck and a 0 to 2 kg load cell. The angle through which the other end of the specimen is turned is measured by taking a drive from the electric motor, through a reduction gear box, to a helically-wound potentiometer. The outputs from the load cell and potentiometer are fed to a recorder, so that (apart from scale factors) a stress-strain characteristic can be plotted. The exact value of the angle of rotation can be checked by an auxiliary counter also driven from the electric motor. There is an anti-backlash bearing on the shaft driving the chuck.

Although tests were made over a range of rotational speeds from about 0.22 to 0.7 ^{c}/min, most runs were made at a rotational speed of 0.3 ^{c}/min.

The composite specimens were 15.2 cm long with the centre 10 cm turned down to a diameter of about 0.62 cm. They were made by wet lay-up technique using type 1, high modulus, and type 2, high strength, carbon fibre and an epoxy resin. The carbon fibres were aligned approximately parallel to the long axis. Some of the fibre of either type was treated by the Harwell process to give an improved adhesion between the resin and fibre. The epoxy resin consisted of 100 parts by weight (pbw) of a liquid bisphenol A resin, 80 pbw of methyl nadic ahydride hardener, and 1 pbw of benzyl dimethylamine accelerator, cured for 3 h at 120° C. The void content of the composite bars was negligible.

A specimen diameter of 0.62 cm was chosen since for diameters much above this the torque required to break the specimen was greater than that which could be measured by the load cell, while for much smaller diameters the torque was so small that significant errors could occur in measuring the slope of the torque/angle of rotation curve.

The fibre volume loadings given on the figures and referred to in the text were calculated from the volumes of fibre used in preparation and the final volume of the specimen. Since no fibre spewage occurred in pressing this gave reasonably accurate values. As a check the fibre volume loadings of some of the specimens were measured by taking a small (0.5 g) sample and digesting the resin with concentrated H_2SO_4 and H_2O_2, washing the fibre obtained, and drying to constant weight at 120° C. The fibre density was measured separately. The results agreed with the calculated ones to within $\pm$ 1.5%. Considerable care should be taken in using the combination of acid and peroxide.

3. Theory

In a circular specimen subject to torsion, there is a shear stress distribution in the plane of the cross section of the specimen and in an axial plane. Since the specimens in the present work were made with axially aligned fibres, shear failure was expected, and did occur, in an axial plane.

Two shear moduli, G_{11} and $G_{\perp}$, can be associated with the composite specimen. G_{11} is the shear modulus in which the fibres are perpendicular to the plane in which the shear force is applied, and $G_{\perp}$ that in which the fibres lie in the plane of the shear force. Using Kittel's [4] notation for the elastic constants, S_{11} etc., $G_{11} = 1/S_{44}$. Hearmon [5] shows that $G_{\perp} = 1/(\frac{1}{2}S_{44} + S_{11} - S_{12})$. The modulus specifically measured here was G_{11}. It will henceforward be referred to as G.

If a circular rod, of diameter d and length l, is clamped at one end and a torque T applied at the other, then it is easy to show, see Timoshenko and Young [6], for instance, that the shear modulus is given by the equation

$$G = \frac{32T}{\theta \pi d^4} \tag{1}$$

where θ is the angle of rotation/unit length.

The maximum shear stress, τ_{max}, occurs at the surface of the rod and provided the torque is linearly related to the angle of rotation up to the point of failure,

$$\tau_{max} = \frac{16T}{\pi d^3} \tag{2}$$

If the shear stress strain characteristic is non-linear, i.e. as shown in Figs. 1 and 2, then a slightly different analysis must be used to obtain τ_{max}. In the first case, Fig. 1, the rate of increase of the torque, T, falls off and then becomes constant with increasing rotation. This can be interpreted as meaning that all the material in a cross section is stressed to the same level, τ_{max}. Thus,

$$T = \int_0^r 2\pi\, a^2\, da\, \tau_{max}$$

and

$$\tau_{max} = \frac{3T}{2\pi r^3} \tag{3}$$

where $r > a > 0$.

If the form of the shear stress strain characteristic is as shown in Fig. 2, then the following method due to Nadai [7] must be used. This depends on the fact that the stress across a section can be regarded as a function of either radius or strain, and the dependence of stress on strain is the same whether different angles of twist are considered at one radius as the test proceeds, or different radii are considered at the same stage in the test. Thus the variable of integration may be changed from radius to shear strain. Now the total torque is given by

$$T = 2\pi \int_0^r \tau(a)\, a^2\, da$$

But the shear strain, γ, is given by $\gamma = a\theta$

$$\therefore T = \frac{2\pi}{\theta^3} \int_0^{\gamma_0} \tau(\gamma)\, \gamma^2\, d\gamma$$

where γ_0 is the maximum shear strain. As $\gamma_0 = r\theta$ the integral is a function of θ and differentiating both sides with respect to θ gives, since $\tau_0 = \tau_{\max}$,

$$\frac{d}{d\theta}(T\theta^3) = 2\pi\, \tau_{\max}\, r^3\, \theta^2$$

but

$$\frac{1}{\theta^2}\frac{d}{d\theta}(T\theta^3) = \theta\frac{dT}{d\theta} + 3T$$

$$\therefore \tau_{\max} = \frac{1}{2\pi r^3}\left(\theta\frac{dT}{d\theta} + 3T\right) \quad (4)$$

Referring to Fig. 2 it can be seen that

$$CS = \frac{\theta dT}{d\theta}$$

and $BS = T$. Obviously if the form of the shear stress strain characteristic is such that $CS = 0$ Equation 4 reduces to Equation 3.

It is simple to make square cross-section bars and it would be preferable to use these without machining. However, though solutions analogous to Equations 1 and 2 exist for non-round cross-section bars, see Timoshenko and Goodier [8] and Love [9], and exact values of G can be obtained easily if the elastic stiffness constants C_{44} and C_{55} are equal, because of the lack of symmetry of non-round cross-sections, it is not possible to correct for the non-linearity of the stress-strain characteristics. Thus accurate values of the shear strength cannot be obtained.

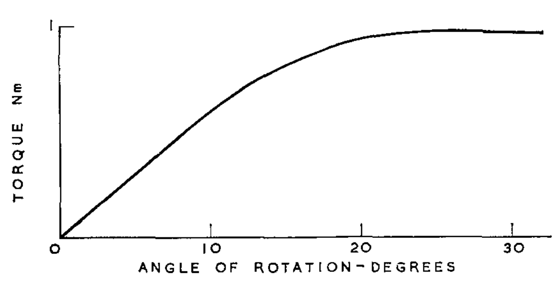

Figure 1 Torque versus angle of rotation for a 1U or pure resin specimen.

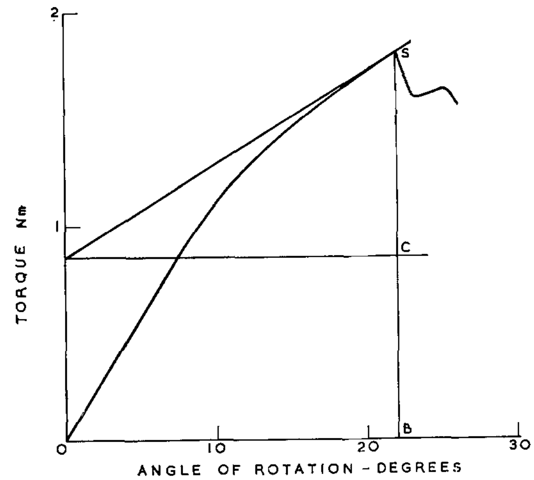

Figure 2 Torque versus angle of rotation for a 1T, 2U, or 2T specimen.

4. Results and discussion

A typical torque twist curve for a composite made from type 1 untreated (1 U) carbon fibre or a pure resin specimen is given in Fig. 1, and for type 1 treated (1T) or type 2 treated or untreated (2T, 2U) carbon fibre in Fig. 2. In both cases the slope and hence shear modulus is initially constant. Eventually in the former case the shear stress, and torque, become constant throughout the specimen, and failure is said to have occurred. With the latter the value of the shear modulus, G, decreases as the angle of rotation is increased, until finally a break occurs at the surface and there is a sudden drop in the torque transmitted by the specimen.

Figs. 3 and 4 show the shear modulus and strength respectively as a function of the volume loading of fibres, for 1T and 2T and 1U and 2U fibre composites. Individual points represent the average of four readings. All measurements were made at a temperature of approximately 25° C. At a given volume loading the shear moduli for composites made from 1T, 2U and 2T fibre tend to separate, the modulus for 2T material being

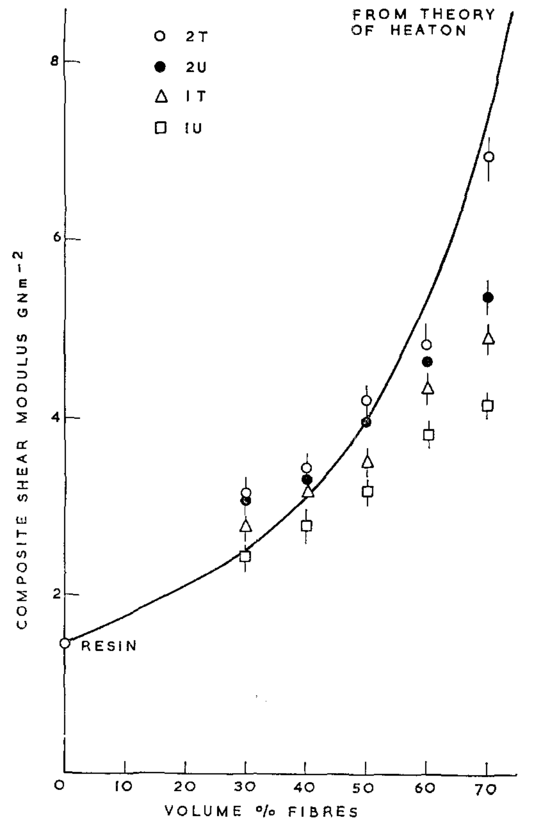

Figure 3 Composite shear modulus versus volume of fibres, for 1U, 1T, 2U, and 2T specimens. The full line is from the theory of Heaton.

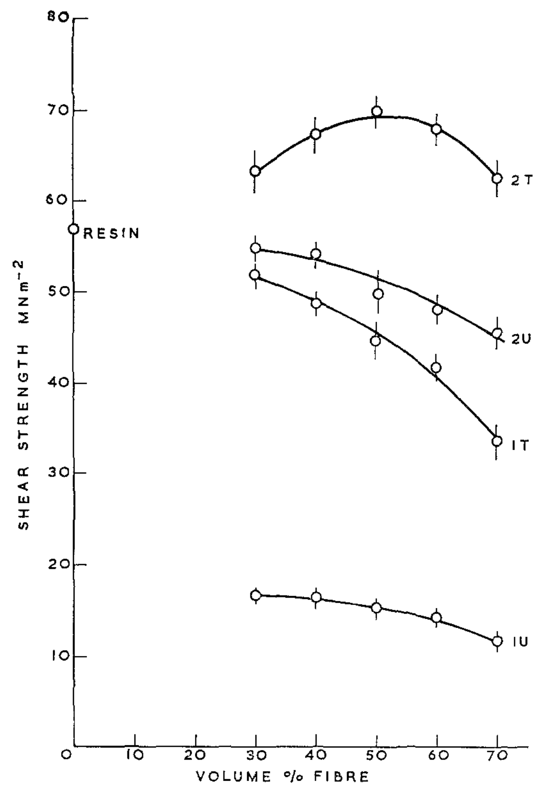

Figure 4 Torsional shear strength versus volume of fibre for 1U, 1T, 2U, and 2T specimens.

the largest and that for 1T the smallest. Except for 30 and 40% volume loadings, the results for 2T composite agree reasonably with the theoretical curve of Heaton [10] derived for a square array of fibres and a fibre-to-matrix shear-modulus ratio of 12:1. It is suggested that at the higher volume loadings values for 2U and 1T fibre composites fall below the theoretical curve, not because of differences in fibre shear modulus, but because of poorer adhesion between the fibre and resin. At the lower volume loadings moduli are probably greater than theoretical values because the matrix deforms plastically, allowing stress to be transferred from the resin to the fibre. All the results for 1U fibre composite, which invariably had a shear stress strain characteristic of the type shown in Fig. 1, lie on a straight line below the other results. In this case adhesion between the resin and fibre is so low that substantial shear deformation probably occurs in the square ends of the specimen gripped in the chucks. This would mean that the value of l used in Equation 1 was too small and hence a low value of shear modulus obtained.

Shear strengths, calculated from Equations 3 and 4 as appropriate, show that there is a considerable difference between the two types of fibre composite and between composites made from treated and untreated fibre. The improvement due to the surface treatment of the fibre is obvious. With the exception of 2T fibre composites, all shear strengths decrease with increasing volume loading of fibre. The shear strength of the former composite is greater than that of the pure resin and has a maximum at about 50% volume loading of fibre. The use of Equation 2, which does not allow for non-linearity in the stress strain characteristic, to calculate the shear strengths, gave values of shear strength up to 20% higher than those shown in Fig. 4.

The coefficients of variation for the torsional shear strength results lie between 2 and 12% with no apparent correlation between a high (or low) coefficient of variation and fibre type, surface condition, or volume loading. An average figure for the coefficient of variation of short beam shear strengths for 60 % volume loading 1T or 2T material is about 5%. Obviously the statistics are much better for the latter case.

It might seem that, as a fibre can be considered as a stress raiser in the resin, the presence of fibres should give composite shear strengths less than those of the resin; particularly above 50% volume loading of fibres, because of the rapid increase of shear-stress concentration between the fibres in this region. This conclusion agrees with some, but not all, of the results.

Reynolds and Hancox [2] have recently shown a better approach to the dependence of shear strength on volume of fibres is provided by the method of fracture mechanics. The theory developed is plausible and, although it does not yet allow the calculation of absolute shear strength, it shows that there is no reason why the shear strength for a composite should not exceed that of the resin.

Briefly, by analogy with conventional fracture mechanics, it is proposed that the bond energy per unit area, G_{2c}, is related to the shear strength and modulus, and the stress concentration due to the presence of the fibres, β, by the equation

$$G_{2c} = \alpha \beta^2 \frac{\tau^2}{G} \quad (5)$$

where α includes stress concentration due to the specimen shape and the value of the critical flaw length. By taking shear strength and modulus results for a pure resin and a 2T fibre composite, and assuming for the latter that the bond between the fibre and resin is sufficiently good that failure occurs within the resin rather than at the interface or in the fibre, values of β can be calculated. These compare sufficiently well with theoretical values, Heaton [10], to indicate that the model is reasonable.

The above approach can be applied to the shear-strength results as follows. For 2T composites the assumption about the bond seems reasonable so that, provided individual values of shear strength and modulus for a given volume loading obey Equation 5 with G_{2c}/α constant, there is no reason why the shear strength of the composite should not exceed that of the pure resin.

Since the values of β should be independent of fibre type or treatment, Equation 5 can be used, with experimental values of shear strength and modulus, to calculate G_{2c}/α for the other composite types. In these cases G_{2c}/α is below the value for 2T fibre, indicating that the bond could be improved, and falls with increasing fibre content. When separate values of G_{2c} and the critical flaw length are available, it will be possible to calculate the shear strength as a function of volume loading.

Small cracks in the surface of the specimen parallel to the long fibre axis were sometimes visible in failed specimens of all types, except those made from 1U fibre, and polished cross-sections of failed specimens sometimes showed a network of fine cracks between fibres. All types of specimen, but particularly those made of 1U fibre, often showed a permanent set, after testing. Interesting behaviour was sometimes noticed with 1U fibre composites. If the test was continued for long enough, after a period in which the torque remained constant, it was found to increase again, very slowly, with twisting, sometimes by as much as 100% before a final catastrophic failure. This is believed to be due to fibres slipping in the resin matrix and then bonding with the resin mechanically. The modulus in this region was very low, well below the initial value.

Failure, indicated either by the onset of plastic behaviour, see Fig. 1, or a sudden drop in the torque transmitted, see Fig. 2, occurred at deflections between 20 and 75°. The deflection for either fibre type was greatest for 30 % volume loading and then with one or two exceptions fell fairly rapidly with increasing fibre content. For any fibre volume loading 2T specimens had the greatest deflection at failure and 1U the least, with 1T and 2U in order in between. However the permanent set after failure was usually greater with 1U specimens, the value for a 30% 1U specimen being about 50°.

Pure resin specimens usually failed by shattering near one of the gripping points.

Adams and Thomas [11] and Novak [12] have also used the solid-rod torsion test to measure shear modulus and strength. The former noted a dependence of shear modulus on fibre volume fraction, similar to that found here, while the latter noted that the shear strength of untreated fibre composite decreased with increasing fibre volume, while that of surface-treated material increased. Novak also noted an increasing amount of tensile failure in higher volume loading treated-fibre specimens. Reynolds [13] has suggested that this was because neither of the chucks in Novak's apparatus could move

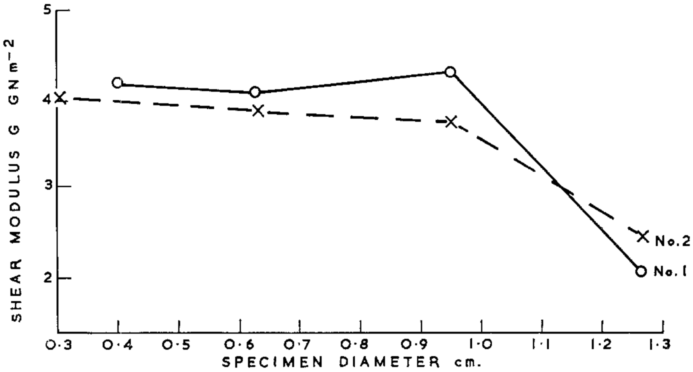

Figure 5 The shear modulus of specimens of a 60% 1T composite as a function of the specimen diameter.

longitudinally, so that the specimens were subjected to a tensile stress as well as a shear one.

Fig. 5 shows the shear modulus as a function of the specimen diameter for two specimens made from 60% of type 1T fibre. The original diameter of both was 1.27 cm. This was reduced to 0.3 or 0.4 cm in steps, the shear modulus being measured each time. The modulus is significantly lower for the largest diameter, but then increases and remains fairly steady as the diameter is reduced. Though the rate of rotation in a torsion test is constant, the strain-rate is proportional to the radius of the specimen. Hence, to check if the effect of diameter ws due to changing strain-rate, the moduli for 1.27 and 0.625 cm diameter specimens were measured at rotational rates of 0.05 and 0.5 c/min. In neither case did the moduli for the same specimen differ by more than 5%, nor could the larger modulus be associated with the faster rate of rotation. It is suggested that the low results for the 1.27 cm diameter specimens are due to a poor fibre distribution in the outer parts of the specimen.

Finally, to investigate the effect of the shape of the cross-section on the shear modulus, measurements were made on a set of eight specimens with a square cross-section and then repeated after the centre 10.0 cm of each specimen has been turned down. The average modulus of the square cross-section specimens was 4.18 ± 0.14 GNm^{-2} and of the same specimens after turning down 4.4 ± 0.09 GNm^2. Thus it appears that for modulus determinations it is possible to work with square cross-section specimens. It should be noted, however, that in a few other cases a difference of up to 15% has been noted between the shear modulus before and after machining the central section. This is again presumably due to the removal of a resin-rich or an otherwise non-typical layer from the surface.

Conclusion

The shear modulus of unidirectional carbon fibre composites made from 2T high-strength fibre increases rapidly with increasing volume loading of fibre and agrees reasonably, except for low volumes of fibre, with the theoretical values of Heaton. Values for 2U and 1T carbon fibre composites do not increase so rapidly with increasing volume fraction, and fall below the theoretical curve at higher volume loadings. Results for 1U fibre composites are below the others, and the theoretical curve, probably because of poor adhesion between the fibre and composite.

Because failure is due to shear forces only, the torsion test, when corrected for the non-linearity of the stress strain characteristic, gives true values of the shear strength irrespective of the volume loading or strength of the fibre. In common with other less accurate tests, the torsion test shows that 2T fibre composites have a better shear strength than type 1 and that treating the fibre surface to improve adhesion improves the shear strength. For composites containing fibres other than 2T, the shear strength decreases steadily with increasing volume of fibre. 2T fibre composites have a shear strength greater than that of the pure resin, with a maximum at about 50% by volume of fibre. When viewed from the point of view of fracture mechanics, this behaviour has been shown to be reasonable.

For measuring shear modulus, square cross-section specimens are, if care is taken, as suitable as round ones.

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